Pragmatic multi-scale and multi-physics analysis of Charles Bridge in Prague

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with
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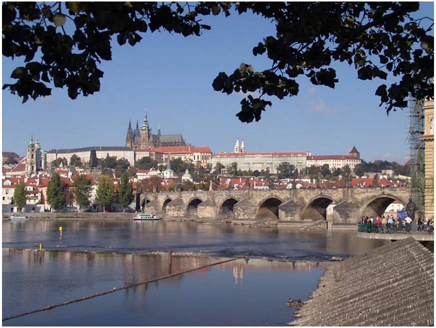
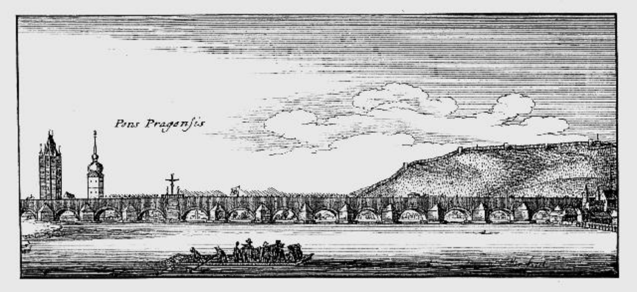
Motivation



Charles Bridge in Prague

1357 Foundation stone laid (9th July 1357, 5:31)
1406 Completion of Charles Bridge
1432 Damage due to flood
1496 Erosion by the flow of the water and pier No. 3 drop
1503 Repair of damage from years 1432 and 1496
1655 Damage to the pier foundations
1784 Damage to the foundation of three piers and five vaults
1788 Repair of damage from year 1784
1890 Vaults No. 5,6 and 7 destroyed, piers No. 4, 8 damaged
1903 Rehabilitation of piers No. 3, 4 and 7
1975 Major reconstruction, reinforced concrete slab installed
2002 More than 100-year flood, the bridge survived

http://www.zastarouprahu.cz/kauzy/kmost/promeny.htm

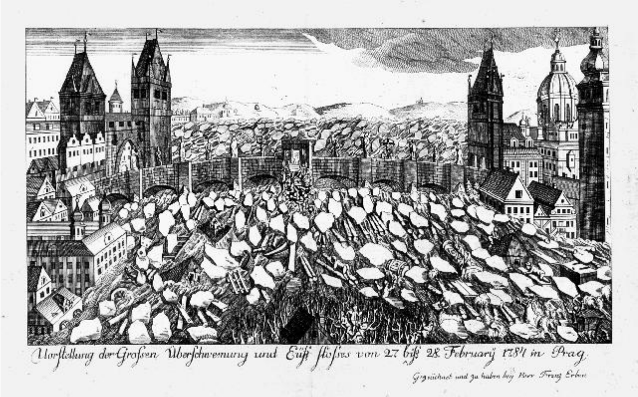


Charles Bridge, 1635

Motivation

Brief historical excursion

http://www.zastarouprahu.cz/kauzy/kmost/promeny.htm



Charles Bridge during flood in 1784

Motivation

Brief historical excursion

http://www.zastarouprahu.cz/kauzy/kmost/promeny.htm

Charles Bridge during flood in 1872

Motivation

Brief historical excursion

http://www.zastarouprahu.cz/kauzy/kmost/promeny.htm

Charles Bridge during flood in 1890

http://www.zastarouprahu.cz/kauzy/kmost/promeny.htm

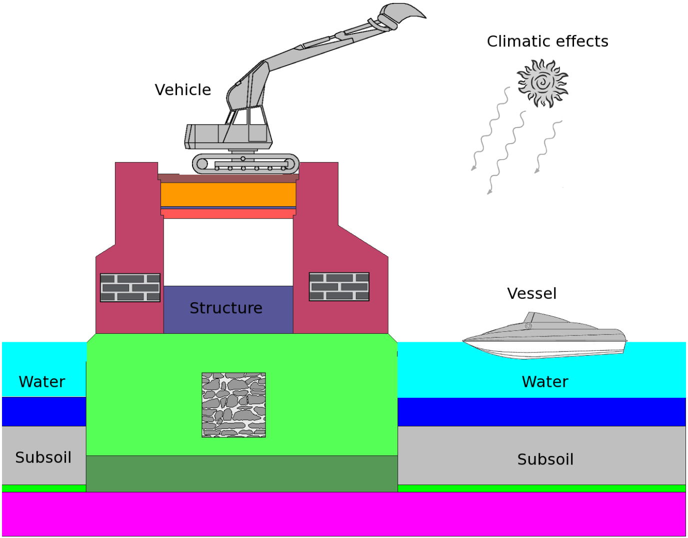
Charles Bridge during flood in 2002

- **1994–2001** Concept of a major repair of the bridge
  - Removal of concrete slab
  - Replacement of all road layers
  - Replacement of filling between the slab and bridge vaults
  - Strengthening of the whole structure
- **2001–2003** Intensive discussion on the proposed concept
- **2003** New concept of a bridge repair required
- **2005** Team headed by Jiří Šejnoha, FCE in Prague contacted for computational assessment of the bridge
- Analysis requirements
  - Three dimensional non-linear mechanical model
  - At least two- and six-spanned segment
  - Mechanical analysis based on established commercial codes
  - Overall time for analysis approximately two months

Outline

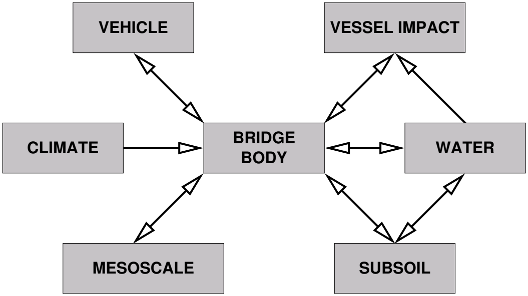
- Modeling strategy
- Nonlinear analysis of masonry structures on mesoscale
  - Application of homogenization based on periodic fields
  - Construction of statistically equivalent periodic unit cell
  - Selecting the representative size of SEPUC
  - Evaluation of effective properties - homogenized fracture energy
- 3D Macroscale simulations - engineering approach
  - Geometrical model
  - Selected loading - thermal effects, water pressure, floating vessel impact

Modeling strategy
Analysis overview



Modeling strategy

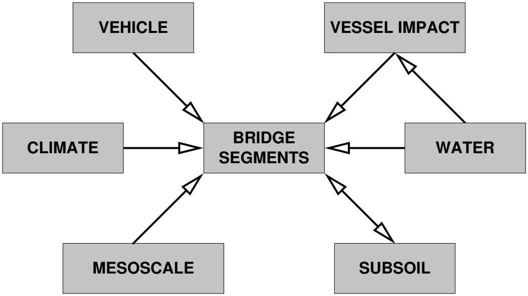
Ideal flowchart



- Fully-coupled
- Multi-scale
- Multi-physics
- Non-stationary
- Three-dimensional
- Currently not feasible

Modeling strategy

Pragmatic flowchart



- Fully-uncoupled: emphasis given to mechanical part of analysis
- Multi-scale: unit cell simulations to feed material models
- Multi-physics: separate analysis for individual phenomena
- Stationary/static analysis
- Three-dimensional mechanical analysis
- Feasible within $\approx$ 2 months

- Different texture for individual parts of structure
  - Regular masonry of vaults
  - Non-regular masonry of parapet walls
  - Filling quarry masonry
- Data for individual components available from experiments
- “Virtual testing” by the 1st-order homogenization

- One-point probability function

$$S_r(x) = P(\chi_r(x = 1))$$

- Two point probability function

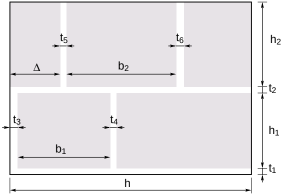
$$S_{rs}(x, y) = P(\chi_r(x)\chi_s(y) = 1)$$

- Ergodicity and statistical homogeneity assumption

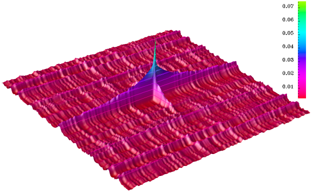
$$S_r = c_r \qquad |\Omega| S_{rs} = \mathcal{F}^{-1}(\widetilde{\chi_r} \cdot \overline{\widetilde{\chi_s}})$$

Original structure

Idealized unit cell



Example of S_{mm}



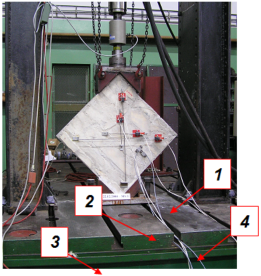
- Objective function - minimization based on genetic algorithms

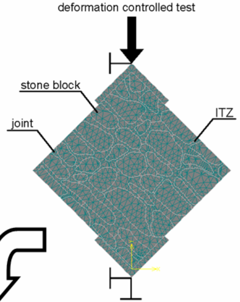
$$E = \sum_i \sum_j (S^0_{rs}(i,j) - S_{rs}(i,j))^2$$

- Small strain plastic fracturing constitutive model
  - Menetrey-Willam yield surface in compression
  - Rankine-type yield criterion in tension
  - Smeared crack model with mesh adjusted softening modulus
    - Energy dissipation is linked to element size

$$\varepsilon^c = \frac{w^c}{h} \leftarrow h = \alpha\sqrt{A_e}$$

- Main input parameters
  - Young's modulus E and Poisson's ratio ν
  - Tensile strength f_t and Fracture energy G_f
  - Interfacial properties





STONE BLOCK & MORTAR

-quasi-brittle material

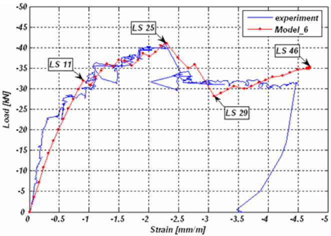
-plastic fracturing material model

ITZ & "JOINT"

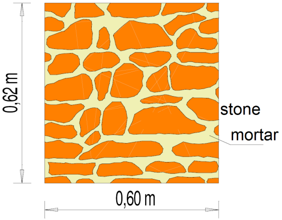
-interface material model

-Mohr-Coulomb criterion with tension cut off

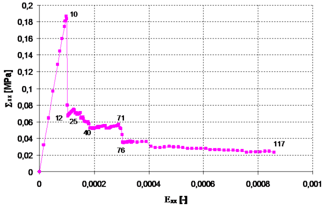
Interfacial material properties
-Cohesion
-Angle of internal friction



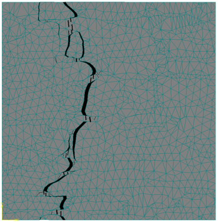
Geometry



Stress-strain curve



Crack pattern



- Homogenized fracture energy - analogy with smeared crack model

$$G_F = \int_0^{W^c} \Sigma \, \mathrm{d}W^c = L \int_0^{E_{\max}} \Sigma \, \mathrm{d}E$$

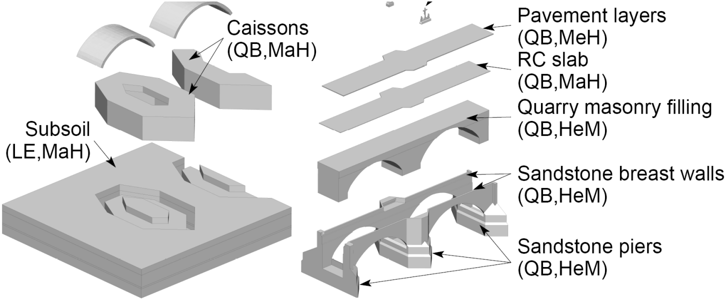
- RILEM-type effective fracture energy estimate

$$G_F \approx \frac{LH}{\ell} \int_0^{E_{\max}} \Sigma \, \mathrm{d}E$$

Macroscopic analysis - geometrical model

Courtesy of Zdeněk Janda

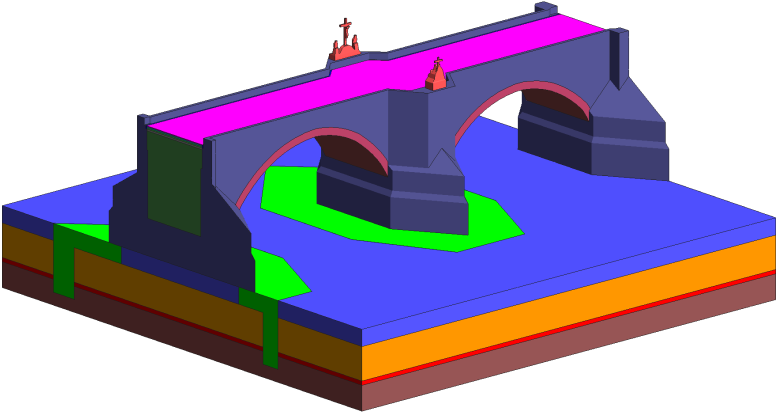
- Based on extensive geodesic three-dimensional data
- Conversion to simplified CAD model
- Decomposition into quasi-homogeneous sub-volumes



- Two- and six-span variants

Macroscopic analysis - geometrical model

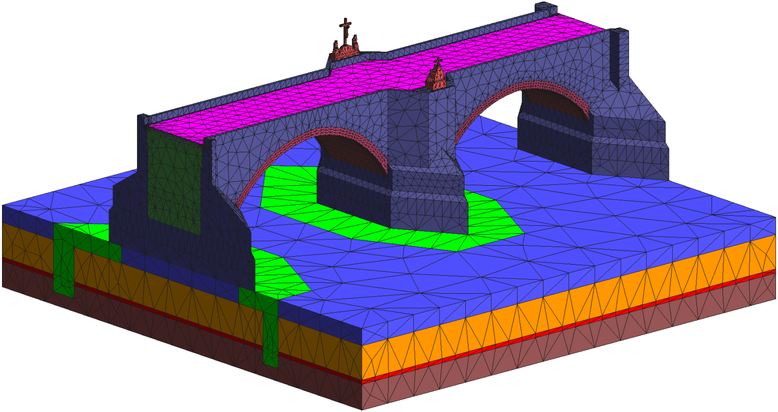
Two-span segment



CAD model

Macroscopic analysis - geometrical model

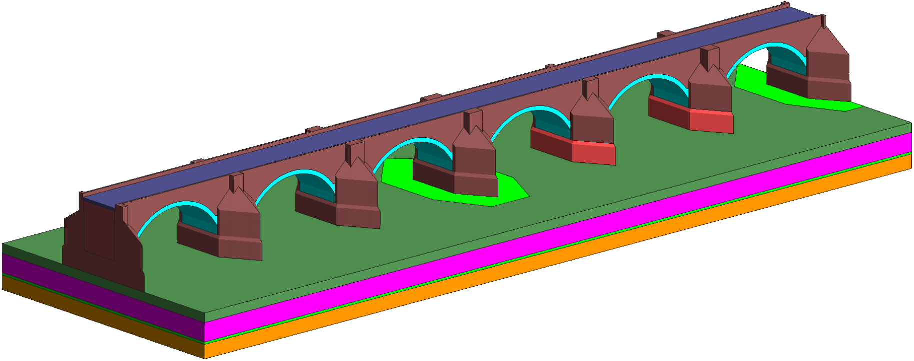
Two-span segment



Finite element mesh (20,409 nodes; 97,004 linear tetrahedra)

Macroscopic analysis - geometrical model

Six-span segment



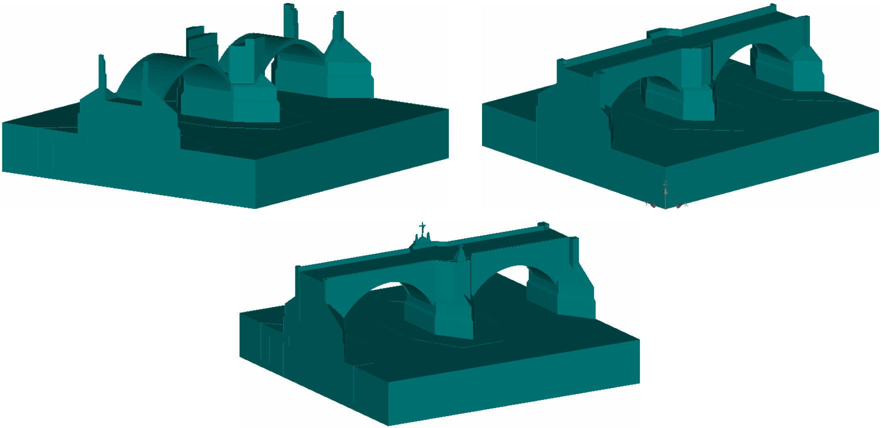
Six-span model (31,725 nodes, 142,976 linear tetrahedra)

- Response to
  - Self weight
  - Temperature impact
  - Hydrostatic and hydrodynamic loading during floods
  - Impact of ice block
  - Impact of a tag boat (2300 t)
- Carrying capacity of the bridge - six-span model
  - Construction vehicles

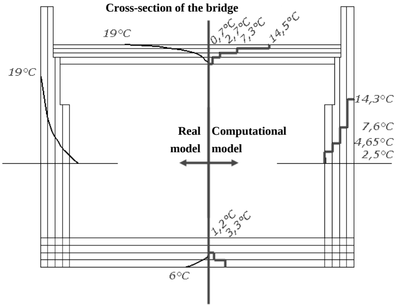
Macroscopic analysis - actions on structure

Self weight

- Dominant permanent action
- Stages of construction need to be modeled properly



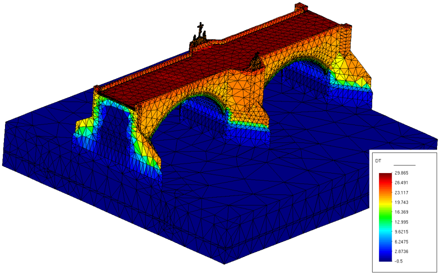
- Two-dimensional non-stationary coupled heat and moisture transfer during a typical year
- Simulation performed in finite volume code DEPLHIN
- Very good correlation with experimental data ($\pm 5^0$C)
- Estimated temperature distribution in winter and summer periods

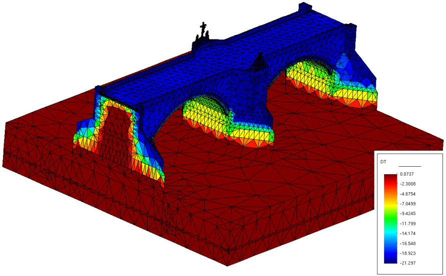


Macroscopic analysis - actions on structure

Temperature change

- Previous analysis gives extremal values of surface and internal temperatures
- Stationary three-dimensional heat transfer analysis with given data
- Input for mechanical model





Macroscopic analysis - actions on structure

Water pressure (Čihák, 2002)

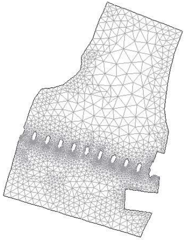
- Hydrostatic pressure

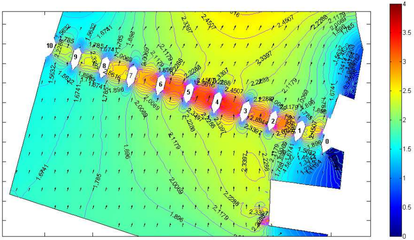
$$p_s(\mathbf{x}) = h(\mathbf{x})\rho_w g$$

- Hydrodynamic pressure

$$p_d(\mathbf{x}) = \frac{1}{2}C(\mathbf{x})\rho_w v_w^2$$

- Water velocity derived from independent flow analysis



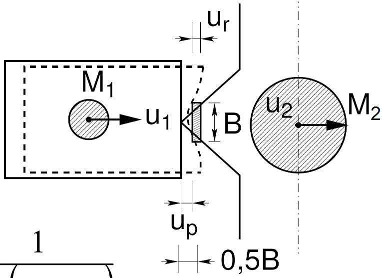


- Impact of a 2300 t tag boat into the bridge
- Simplified two-degree of freedom model
- Number of uncertainties (boat and bridge compliances, energy dissipation during impact)
- Parametric study leading to a conservative estimate

$$u_1(t) = u_r(t) + u_p(t) + u_2(t)$$

$$\ddot{R}(t) + \omega_1^2 R(t) = \frac{-1}{c_r + c_p}\ddot{u}_2(t)$$

$$\ddot{u}_2(t) + \omega_2^2 u_2(t) = \frac{R(t)}{M_2}$$



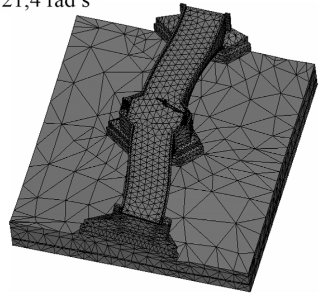
Effective natural frequency $\omega_1^2 = \frac{1}{M_1(c_r + c_p)}$

Macroscopic analysis - actions on structure

Vessel impact

Eigenmode corresponding to

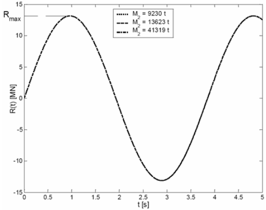
$\omega_2 \approx 21{,}4 \text{ rad s}^{-1}$



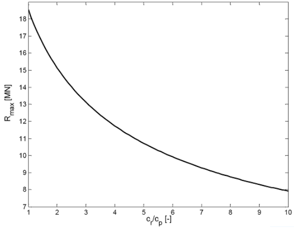
Final comment

The resulting equivalent static force ranging from 8MN to 18MN is comparable to loading due to water pressure for typical flood conditions. The selected value of 12.5 MN is essentially 5x more than the force attributed to the impact of an ice block

Time variation of contact (impact) force



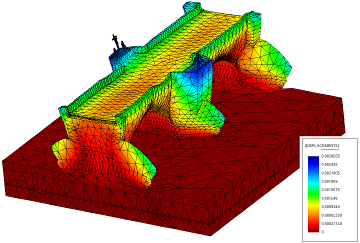
Variation of R_{max} **with the** c_r / c_p **ratio**



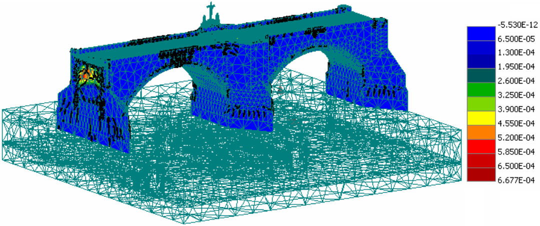
Analysis results - thermal effects

Dead load ⊕ Summer temperature ⊕ Elevated water

- Deformed shape of the structure



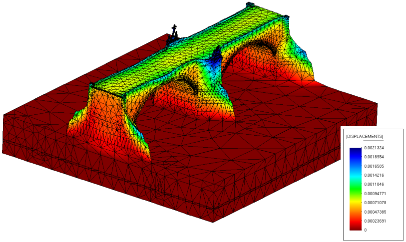
- Distribution of cracks Mode I crack opening displacement



Analysis results - thermal effects

Dead load ⊕ Winter temperature ⊕ Elevated water

- Deformed shape of the structure



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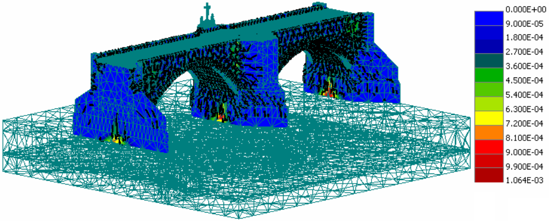
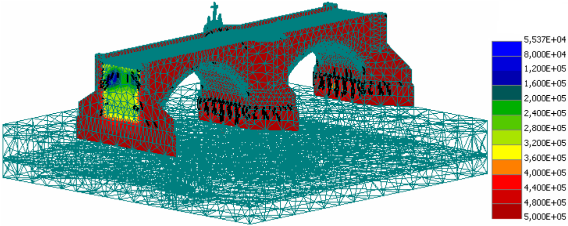


Analysis results - thermal effects

Dead load ⊕ Summer temperature ⊕ Winter temperature ⊕ Elevated water

- Residual tensile strength



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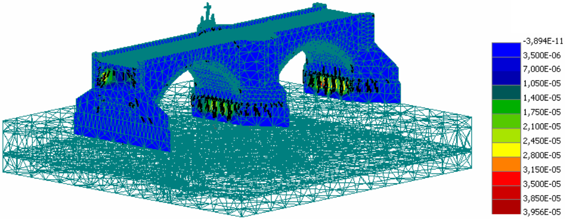
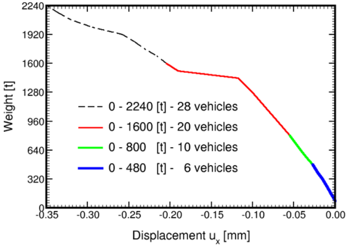
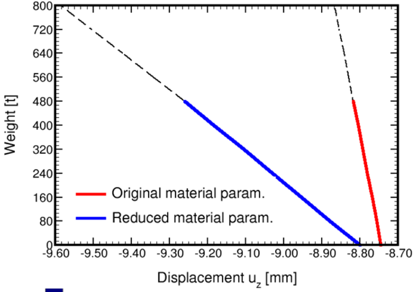


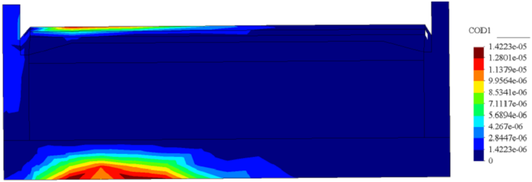
Horizontal displacement – parapet wall



Vertical displacement – arch crest



Distribution of cracks in transverse cross-section



Summary and conclusions

- Theoretical outcomes
  - Simplified ($\equiv$ un-coupled) multi-scale and multi-physics approaches fully capable of providing valuable data for practical engineering problems
  - Homogenization approaches allow for partial replacement of experimental procedures
  - Predicted damage of the structure corresponds well to in-situ observations
- Practical outcomes
  - The structure proved to be stable for load combinations both globally and locally
  - Temperature load seems to be the most severe loading case in terms of extent of corresponding damage
  - The load-bearing capacity of the bridge governed by pavement characteristics
  - Foundations are the most critical part of the structure (currently repaired)
  - Only minor reconstruction operations needed